Christ Church Grammar School

2019
TEST 3

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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Instructions to candidates

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3. Answer all questions.

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6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(7 marks)**

Determine the following.

(a) $\int e^{7x+3}dx$ (1 mark)

(b) $\frac{d}{dx}(x^2e^{x^2})$ (2 marks)

See next page

Question 1 continued

Evaluate the following.

(c) $\int_0^2 3(x + e^{3x})dx$ (2 marks)

(d) $\int_0^{\frac{\pi}{2}} \frac{d}{du} sin(u)du$ (2 marks)

See next page

Question 2 **(3 marks)**

For a $5 monthly fee, a TV repair company guarantees customers a complete service. The company estimates the probability that a customer will require one service call in a month is 0.05, the probability of 2 calls is 0.01 and the probability of 3 or more calls is 0.00. Each call costs the repair company $40.

(a) Complete the table below. (1 mark)

Calls	0	1	2	≥3
Gain (g)			-75	
P(G = g)		0.05		0.00

(b) What is the TV repair company's expected monthly gain from such a contract?
(2 marks)

Question 3 **(5 marks)**

(a) Determine $\frac{dy}{dx}$ where $y = xe^{2x-1}$ (2 marks)

(b) Hence, determine $\int xe^{2x-1}\,dx$ (3 marks)

End of questions

Additional working space

Question number: _____

See next page

Additional working space

Question number: _____

Additional working space

Question number: _____

Additional working space

Question number: _____

Christ Church Grammar School

2019
TEST 3

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

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Formula Sheet (retained from Section One)

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See next page

Question 4 **(4 marks)**

Assume one in 300 Australian adults were plumbers. One Australian adult was randomly selected and it was noted whether he/she was a plumber. Define X as the random variable associated with this trial.

(a) Describe the distribution of X, include its parameter(s). (2 marks)

(b) State the mean and variance of this distribution. (2 marks)

See next page

Question 5 **(6 marks)**

The table shows the pdf of a discrete random variable, where $E(X)$ is the expected value of X.

x	1	2	3	4	5
$P(X = x)$	0.2	p	0.3	q	0.1

(a) If $E(X^2) = 8.2$, determine p and q. (3 marks)

(b) Calculate $E(X)$. (1 mark)

(c) If $Y = 1 - 2X$, calculate $E(Y)$. (2 marks)

See next page

Question 6 **(5 marks)**

The population of a certain country is growing continuously at 3% per annum. Its population P is such that $P = P_0 e^{kt}$ where P is the population in millions, t years from now. The population is currently 35 million.

(a) In how many years will the population of the country reach 50 million if it continues to grow at the same rate?

(2 marks)

(b) Data suggests that the capital city's population is growing at a faster rate than that of the country. Currently 22% of the people in the country live in the capital city, and if its population continues to grow at its present rate, 40% of the entire population will live in the capital city 15 years from now. What is the continuous growth rate of the population of the capital city? (3 marks)

See next page

Question 7 **(5 marks)**

(a) A coin is biased in favour of heads such that the probability of obtaining a head on any single toss is 0.6. The coin is tossed three times and the result noted. If X is the number of heads obtained on the three tosses, find $E(X)$, the expected value of X by first completing the table below: (3 marks)

x	0	1	2	3
$P(X = x)$				

(b) For the random variable X defined above, find:

(i) $E(3X + 1)$

(1 mark)

(ii) $SD(3X + 1)$

(1 mark)

See next page

Question 8 **(4 marks)**

The Kappa family live one kilometre from their school. On the route they drive to school are two school crosswalks. The probability that they will have to stop at each crosswalk is $\frac{3}{5}$ and is independent of each other. Let x represent the number of times the car must stop at a crosswalk.

(a) Complete the following probability distribution table for this random variable. (2 marks)

x	0	1	2
$P(X = x)$			

(b) After five weekdays, what is the probability that the Kappa family have to stop at most once on their way to school on exactly three of the five days? (2 marks)

See next page

Question 9 **(3 marks)**

Find the coordinates of the point(s) on the curve $y = x + e^{2x}$ where the tangent to the curve at these point(s) are parallel to $3x - y = 1$.

Question 10 **(3 marks)**

A bush fire near Walpole at time t hours is spreading at a rate of $2.1e^{2t-6}\ m^2/hour$.

(a) What area is burnt out in the first 9 hours? (1 mark)

(b) What area is burnt out during the 11th hour? (1 mark)

(c) Explain why this function is an unrealistic model for $t > 11$. (1 mark)

End of questions

Additional working space

Question number: _____

Additional working space

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MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name • SOLUTIONS •

Teacher's name_____

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See next page

Question 1 **(7 marks)**

Determine the following.

(a) $\int e^{7x+3}dx$ (1 mark)

$= \frac{1}{7}e^{7x+3} + c$ ✓

(b) $\frac{d}{dx}(x^2 e^{x^2})$ (2 marks)

$u'v + uv'$

$= \underset{\checkmark}{2x e^{x^2}} + \underset{\checkmark}{x^2(2x e^{x^2})}$

OR $2x e^{x^2}(1 + x^2)$

3

Question 1 continued

Evaluate the following.

(c) $\int_0^2 3(x + e^{3x})dx$ (2 marks)

$3\int_0^2 x + e^{3x}\,dx$

$= 3\times\left[\frac{x^2}{2} + \frac{e^{3x}}{3}\right]_0^2$ ✓

$= 3\times\left[2 + \frac{e^6}{3} - (0 + \frac{1}{3})\right]$

$= 5 + e^6$ ✓ ✓

(d) $\int_0^{\frac{\pi}{2}} \frac{d}{du}\, sin(u)du$ (2 marks)

$= \left[\sin u\right]_0^{\pi/2}$ ✓

$= \sin \pi/2 - \sin 0$

$= 1$ ✓ ✓

4

Question 2 **(3 marks)**

For a $5 monthly fee, a TV repair company guarantees customers a complete service. The company estimates the probability that a customer will require one service call in a month is 0.05, the probability of 2 calls is 0.01 and the probability of 3 or more calls is 0.00. Each call costs the repair company $40. What is the TV repair company's expected monthly gain from such a contract?

Begin by filling out the table below.

Calls	0	1	2	≥3
Gain (g)	5	-35	-75	
P(G = g)	0.94	0.05	0.01	0.00

✓ (all correct)

$E(G) = 5(0.94) - 35(0.05) - 75(0.01)$

$= 4.70 - 1.75 - 0.75$ ✓ (working)

$= \$2.20$ Expected Gain/Month. ✓ (Answer)

3

Question 3 **(5 marks)**

(a) Determine $\frac{dy}{dx}$ *where* $y = xe^{2x-1}$ (2 marks)

$u \quad v$ ✓ $\quad u'v + uv'$

$$\frac{dy}{dx} = 1\left(e^{2x-1}\right) + x(2)\left(e^{2x-1}\right) \quad ✓$$

$$= e^{2x-1} + 2x\,e^{2x-1} \quad ✓$$

(b) Hence, determine $\int xe^{2x-1}\,dx$ (3 marks)

$$\frac{d}{dx}\,x\,e^{2x-1} = e^{2x-1} + 2x\,e^{2x-1} \quad ✓ \text{ (Writes statement)}$$

$$\int \left(\frac{d}{dx}\,x.e^{2x-1}\right)dx = \int e^{2x-1}\,dx + \int 2x.e^{2x-1}\,dx. \quad \text{(Uses FTC)} \checkmark$$

$$x\,e^{2x-1} + c_1 = \frac{1}{2}\int 2\,e^{2x-1}\,dx + \int 2x\,e^{2x-1}\,dx.$$

$$x\,e^{2x-1} + c_1 = \frac{1}{2}e^{2x-1} + c_2 + 2\int x.e^{2x-1}\,dx.$$

$$x\,e^{2x-1} + c_1 - \frac{1}{2}e^{2x-1} - c_2 = 2\int x.e^{2x-1}\,dx$$

$$\therefore \int x.e^{2x-1}\,dx = \frac{1}{2}\left(x.e^{2x-1} - \frac{1}{2}e^{2x-1}\right) + c$$

$$\int x.e^{2x-1}\,dx = \frac{x}{2}e^{2x-1} - \frac{1}{4}e^{2x-1} + c. \quad \text{(Correct Answ)} \checkmark$$

End of questions

5

Additional working space

Question number: _____

See next page

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Grammar School

2019
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MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name ° SOLUTIONS °

Teacher's name _____

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Question 4 **(4 marks)**

Assume one in 300 Australian adults were plumbers. One Australian adult was randomly selected and it was noted whether he/she was a plumber. Define X as the random variable associated with this trial.

(a) Describe the distribution of X. Include its parameter. (2 marks)

X is Bernoulli, parameter $p = \frac{1}{300}$ ✓ ✓

(b) State the mean and variance of this distribution. (2 marks)

$E(X) = \frac{1}{300}$ ✓

$Var(X) = \frac{1}{300}\left(1 - \frac{1}{300}\right)$

$= \frac{299}{90000}$ ✓ $(0.00332\dot{}\,)$

4

Question 5 **(6 marks)**

The table shows the pdf of a discrete random variable; where $E(X)$ is the expected value of X.

x	1	2	3	4	5
$P(X = x)$	0.2	p	0.3	q	0.1

(a) If $E(X^2) = 8.2$, determine p and q. (3 marks)

① $p+q = 0.4$ ✓

② $1^2(0.2) + 2^2(p) + 3^2(0.3) + 4^2(q) + 5^2(0.1) = 8.2$

or $0.2 + 4p + 2.7 + 16q + 2.5 = 8.2$ ✓

$\therefore$ $p = 0.3$ $q = 0.1$ ✓

(b) Calculate $E(X)$. (1 mark)

$E(X) = 0.2 + \overset{2p}{0.6} + 0.9 + \overset{4q}{0.4} + 0.5$

$E(X) = 2.6$ ✓

(c) If $Y = 1 - 2X$, calculate $E(Y)$. (1 mark)

$E(Y) = 1 - 2(2.6)$ ✓

$E(Y) = -4.2$ ✓

6

Question 6 **(5 marks)**

The population in a certain country is growing continuously at 3% per annum. Its population P is such that $P = P_0 e^{kt}$ where P is the population in millions, t years from now. The population is currently 35 million.

(a) When will the population of the country reach 50 million if it continues to grow at the same rate? (2 marks)

$P = P_0 e^{kt}$

$P = 35 e^{0.03t}$

$50 = 35 e^{0.03t}$ ✓

$t = 11.889$ ✓

OR Pop^{L} reaches 50 mill ~ 11yrs & 11 months.

(b) Data suggests that the capital city's population is growing at a faster rate than that of the country. Currently 22% of the people in the country live in the capital city, and if its population continues to grow at its present rate, 40% of the entire population will live in the capital city 15 years from now. What is the continuous growth rate of the population of the capital city? **(3 marks)**

✓

15 years time $\Rightarrow 35e^{15\times 0.03}$ (country)

15 years time $\Rightarrow (0.22)\times 35e^{15\times c}$ (city) [where c is growth rate]

$\therefore (0.22)\times 35e^{15c} = 0.4\times 35e^{15\times 0.03}$ ✓ (equation)

$\therefore c = 0.06985$

ie ~ 7% Growth rate. ✓ (rate)

5

See next page

Question 7 **(5 marks)**

(a) A coin is biased in favour of heads such that the probability of obtaining a head on any single toss is 0.6. The coin is tossed three times and the result noted. If X is the number of heads obtained on the three tosses, find $E(X)$, the expected value of X by first completing the table below: (3 marks)

x	0	1	2	3
$P(X = x)$	0.064	0.288	0.432	0.216

✓✓

$(0.4)^3$ $(0.6)(0.4)^2$ $(0.4)^2(0.6)$ $(0.4)(0.6)(0.4)$ $(0.6)^3$

$E(X) = 0.288 + 2(0.432) + 3(0.216)$

$E(X) = 1.8$ ✓

(b) For the random variable X defined above, find:

(i) $E(3X+1)$ (1 marks)

$E(3X+1) = 3(1.8) + 1$

$= 6.4$ ✓ (value)

(ii) $SD(3X+1)$ (1 mark)

old $\sigma_x = 0.8485$

$SD(3X+1) = (3 \times \text{old } \sigma_x)$

$= 2.5455$ ✓ (value)

5

See next page

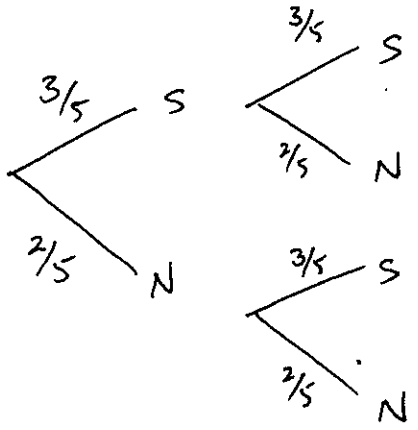
Question 8 **(4 marks)**

The Kappa family live one kilometre from their school. On the route they drive to school are two school crosswalks. The probability that they will have to stop at each crosswalk is $\frac{3}{5}$ and is independent of each other. Let x represent the number of times the car must stop at a crosswalk.

(a) Complete the following probability distribution table for this random variable. (2 marks)

x	0	1	2
$P(X = x)$	0.16 $\frac{4}{25}$	0.48 $\frac{12}{25}$	0.36 $\frac{9}{25}$

(-1 for each wrong) ✓✓



SS : $\frac{3}{5} \times \frac{3}{5} = 0.36$

SN : $\frac{3}{5} \times \frac{2}{5} = 0.24$

NS : $\frac{2}{5} \times \frac{3}{5} = 0.24$

NN : $\frac{2}{5} \times \frac{2}{5} = 0.16$

(b) After five weekdays, what is the probability that the Kappa family have to stop at most once on their way to school on exactly three of the five days? (2 marks)

$P(X \leq 1) = 0.64$ ✓

$\therefore \binom{5}{3}(0.64)^3(0.36)^2 = 0.3397$ ✓

See next page

4

Question 9 **(3 marks)**

Find the coordinates of the point(s) on the curve $y = x + e^{2x}$ where the tangent to the curve at these point(s) are parallel to $3x - y = 1$. → $y = 3x - 1$ $(m = 3)$

Find $\frac{dy}{dx} = 3$

$1 + 2e^{2x} = 3$

$x = 0$

(Show $\frac{dy}{dx} = 3$ ✓ where $\frac{dy}{dx} = 1 + 2e^{2x}$ ✓)

∴ pt $(0, 1)$ ✓

Question 10 **(3 marks)**

A bush fire near Walpole at time t hours is spreading at a rate of $2.1e^{2t-6}\ m^2/hour$.

(a) What area is burnt out in the first 9 hours? (1 mark)

$\int_0^9 2.1e^{2t-6}\,dt = 170\,892.53\ m^2$ ✓

(b) What area is burnt out during the 11th hour? (1 mark)

$\int_{10}^{11} 2.1e^{2t-6}\,dt = 8\,067\,681.55\ m^2$ ✓

(c) Explain why this function is an unrealistic model for $t > 11$. (1 mark)

The function increases too quickly! ✓

End of questions

6

Additional working space

Question number: _____